Principal Varieties of Finite Congruences

Ville Piirainen

The classification of formal languages is one of the central problems in automata theory. For string languages the basic theory is quite well-established, but due to an exponential increase in complexity and the diverse uses of trees in many applications, there are still many questions in the classification theory for tree languages which are open for debate and discovery. Our view of trees and tree languages is strictly algebraic, and we are mainly interested in classes of tree languages of combinatorial nature; for example the varieties of the definite, the reverse definite, or the piecewise testable tree languages.

The general pattern of constructing a variety of tree languages consists of defining a chain or otherwise ordered set of suitable congruences of term algebras whose congruence classes then generate the languages. This set of congruences can be regarded as the generating set of the class, but proving the variety properties of this class using only the basic definitions can be relatively tedious compared to the fact that the result usually is completely obvious. However, each congruence by itself generates a principal variety of finite congruences, and we have developped a few simple criteria which guarantee that if the congruences satisfy them, the union of these principal varieties, which is the same set that the congruences generate together, is indeed a variety. Hence, we can omit many of the technicalities we would face using the original definitions of varieties.

Given a class of tree languages, in addition to the corresponding class of congruences of term algebras, we are also interested in the class of algebras or tree recognizers which recognize the class. The class of such algebras corresponding to a principal variety of finite congruences has an interesting special property: it is the finite part of a locally finite variety of algebras. This correspondence is actually bijective, and might provide interesting new results on tree language varieties.